Preservice teachers' mathematical knowledge of fractions

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Abstract

The main purpose of this study was to assess Taiwanese preservice teachers' mathematics knowledge of fractions. 47 pre-service teachers enrolled in a 4-year teacher education program participated in the Fraction Knowledge Test (FKT) and the Mathematics Problem Solving Ability Scale (MPSAS). Results showed that preservice teachers displayed better fraction knowledge on procedure than on conception. Their fraction procedural knowledge moderately correlated with their problem solving ability. Findings indicated the preservice teachers need more stimuli to construct their conceptual knowledge.

Keywords: preservice teachers, mathematical knowledge, fraction

Introduction

Several researches have revealed that preservice teachers' understanding of fraction content knowledge is very weak (Davis & Thipkong, 1991; Simon, 1993; Behr, Khoury, Harel, Post, & Lesh, 1997; Cramer, Post, & del Mas, 2002). Evidence has shown that preservice teachers have difficulties with the concept of fractions and the meaning of division of fractions (Ball, 1990), cannot understand the operator construct of rational number (Behr et al., 1997), have difficulty in explaining fractions to children and why algorithms work (Selden, & Selden, 1997, Chinnappan, 2000), and can not carry out fractional computation procedures correctly, even when they have correct answers (Becker & Lin, 2005).

Preservice teachers' poor performance in fractions can cause serious problems. Preservice teachers will be teaching mathematics in elementary schools or special education programs. It is substantively important issue and should be addressed.

Why are fractional numbers difficult for many students and preservice teachers? Vergnaud (1983) argued fractional number skill development is heavily dependent on others' essential elements. Fractional numbers are quite different from the whole numbers. Under different contexts, fractional numbers have different personalities. Fractional numbers have different constructs, i.e. part-whole, quotients, measures, ratio, rate, and operators (Behr, Lesh, Post, & Silver, 1983). A part-whole relationship is the description of how much a quantity is relative to a specified unit of that quantity. A quotient is seeing a fraction number as a result of division. A measure is seeing a fraction as a point on a number line. The fraction as ratio is seeing a fraction as a multiplicative comparison between two quantities. A rate is seeing a fraction as a new quantity as a relationship between two other quantities. Finally, an operator is seeing a fraction as a transformation or as a function for another numbers (Ohlsson, 1988; Lamon, 1999; Steefland, 1987).

Another reason for preservice teachers' difficulty with fractions might be due to their poor problem solving ability. Problem solving ability referred to students' ability to solve non-routine mathematics problem (Liu, 1993). As Niemi (1996) indicated, there was a closer association between students' level of problem solving ability and their fraction knowledge. Thus, research focusing on the relationship between preservice teachers' problem solving ability and their fraction knowledge would appear to be necessary.

Recently, teaching students to understand what they are learning has been the major current in mathematics (National Council of Teachers of Mathematics, NCTM, 2000). A similar trend is also emphasized in Taiwan. Due to the Ministry of Education's curriculum standards in 1994, the foci of elementary mathematics learning have been moved from practicing computation skills to encouraging students to develop their own mathematical understandings (Liu, 2000; Tan, 1996; Tsai, 1997). The shift from computation to understanding is expected to balance the weights of procedure and content knowledge in elementary mathematics teaching. In fact, both computational fluency and mathematical understanding are expected to play important roles in the Taiwanese national curriculum standards (Cheng & Lin, 2004).

As Eisenhart et al., (1993) indicated, procedural knowledge refers to the mastery of computational skills and knowledge of procedures in identifying mathematical components, algorithms, and definitions; however, conceptual knowledge refers to the underlying structural relationships of mathematics and the interconnections of ideas that explain and give meaning to mathematical procedures. Many researchers indicted that procedural and conceptual knowledge are both important components of mathematical understanding (Wearne & Hiebert, 1988; Desimone, Smith, Hayes & Frisvold, 2005; Hiebert et al., 2005; Hu & Lee, 2005; Wu & Huang, 2003; Lin & Tsai, 2006). Therefore, both types of knowledge need to be balanced and emphasized when teachers teaching mathematical understandings in fraction.

Purpose

The purpose of this study was to assess Taiwanese preservice teachers' mathematics knowledge in fractions, including their understanding and computational abilities. Three questions were posted (1) How do preservice teachers perform in fractional mathematics knowledge (PK & CK) and problem solving (PS)? And how they correlated with each other? (2) Do the preservice teachers perform equally in fractional procedural and conceptual knowledge? (3) How do they perform in the eight components of fraction in procedural and conceptual knowledge, respectively?

Method

Participants

Participants consisted of 47 pre-service teachers enrolled in a 4-year teacher education program at National Chiayi University in central Taiwan. Although these pre-service teachers had already passed the college entrance examination held in Taiwan, they need to apply for admission before being enrolled in the teacher education program. They are also required to take 5 credit hours in mathematics education, 35 professional education courses, and six months internship before entering elementary school to teach mathematics. Among these 47 participants, there were about 94% of the participants younger than age 25, and approximately 87% were female.

Instrument

Two instruments were used in this study. The first, Fraction Knowledge Test (FKT), was a test of fraction concepts adapted from Cramer, Post, and del Mas's study (2002). It was modified to provide more emphasis on both procedural and conceptual knowledge. An item exemplified as 'Solve the problem $\frac{3}{4}\times\frac{2}{3}$ = ?" is deemed as a procedural knowledge item; another item exemplified as 'Explain how you determined your answer by giving an illustration or representation for $\frac{3}{4}\times\frac{2}{3}$ =?" is deemed as a conceptual knowledge item. The test consisted of 32 items and was specifically designed to measure fraction knowledge in areas related to: (1) CON: concept, (2) EQU: equivalence, (3) ORD: order, (4) ADD: addition, (5) SUB: subtraction, (6) MUL: multiplication, (7) DIV: division, and (8) TRN: transfer. The survey instruments were piloted with 25 pre-service teachers who were enrolled in mathematics methods courses in U.S.A. The instrument was translated into Chinese by one U.S. and one Taiwanese professor, both fluent in Chinese and English. The Chinese version was administered to pre-service teachers in Taiwan during the first week of class in September. The internal reliability (Cronbach's alpha) of the test was found to be .86 in this study.

The second test used was the Mathematics Problem Solving Ability Scale (MPSAS) developed by Liu (1989). There were 16 items (64 sub-questions totally) used in this study. With great reliability (KR20 = .85; equivalent coefficient = .86) and validity (correlation coefficients between MPSAS scores and criterion-related mathematics subjects score, IQ test scores, mathematics diagnostic tests were equal to .77, .72, and .81, respectively), the MPSAS was used to assess these preservice teachers' problem solving ability.

Results and Discussions

Q1: Correlation

As can be seen in Table 1, preservice teachers performed a moderate correlation between PK and CK ($r = .36$, $p < .05$), between PS and PK ($r = .43$, $p < .01$), but had no significant correlation between PS and CK ($r = .002$, n.s.).

As Niemi (1996) indicated, students' problem solving ability correlated with their fraction knowledge, but now we know in advance that the association was with fractional procedural knowledge. This is surprise because problem solving ability referres to a general ability for solving non-routine problems, it might be deemed as a general mathematics potential and should be associated with fractional knowledge. But why now it just related to fractional procedural knowledge, rather to fractional conceptual knowledge?

Table 1

Mean, Standard Deviation, and Correlation for PK, CK, and PS.

	N	M	SD	1	2	3
1PK	47	27.28	3.72	1.0		
2CK	47	19.64	8.77	.36*	1.0	
3PS	44	56.55	4.02	.43**	.002	1.0

*$p < .05$, two tailed.

** $p < .01$, two tailed.

One of reasons might be that our preservice teachers possessed high quality of mathematics knowledge (56.55 out of 64). But this superiority was based on their computation skills (27.28 out of 32), not based on their understandings on underlying structural relationships of mathematics (19.64 out of 32). As a further exploration, we also found PS significantly correlated with the procedural fraction ORD scale ($r = .32$, $p < .01$) and the procedural fraction ADD scale ($r = .36$, $p < .01$), respectively.

Another reason might be due to the formations of MPSAS and FKT. The FKT asked participants to explain why their answer was in a very open question from. But the MPSAS used closed multiple-choice questions to ask participants to explain their understandings. This indicates that preservice teachers in Taiwan know how to compute with fractions but did not understand the rationale behind fractional computation.

Q2: Difference between PK and CK

After a repeated-measure analysis was conducted, the authors found that preservice teachers demonstrated superiority on PK over CK ($F(1, 46) = 40.84$, $p < .001$). This difference can be explained half ($\eta^2 = .47$) from the contribution of mean difference between PK ($M = 27.28$) and CK ($M = 19.64$). Although a balanced emphasis between procedural knowledge and conceptual knowledge had been initiated from 1994, preservice teachers still demonstrated greater procedural than conceptual thinking. This might be due to their early education and training. This also reinforces the lack of conceptual knowledge in fractions for these preservice teachers.

Q3:Performances on components

As can be seen in Table 2, preservice teachers performed differently in the eight components of procedural fraction knowledge ($F(7, 322) = 15.89$, $p < .001$). This difference can be attributed small to the mean differences of the eight fraction components ($\eta^2 = .26$). After conducting least-significant difference (LSD) comparisons, two categories of components were found in fraction procedural knowledge, i.e., Category 1 (ADD, SUB, MUL, DIV) and Category 2 (CON, EQU, ORD, TRN). In Category 1, the components of fractional addition, fractional subtraction, fractional multiplication, and fractional division referred to an algorithmic operation. Category 2, including fractional concept, equivalence, order, and transfer, referred to general attributes of fraction. The preservice teachers

Table 2 - Mean and Standard Deviation for eight components in fraction

	CON	EQU	ORD	ADD	SUB	MUL	DIV	TRN
PK: $F(7, 322) = 15.89$, $p < .001$, $\eta^2 = .26$)								
M	3.32	2.49	2.85	3.79	3.91	3.64	3.87	3.40
SD	.96	1.27	1.23	.86	.58	.76	.49	1.10
CK: $F(7, 322) = 7.86$, $p < .001$, $\eta^2 = .15$)								
M	3.09	2.34	2.34	2.79	2.47	2.19	1.55	2.87
SD	1.16	1.46	1.52	1.61	1.61	1.71	1.74	1.47

performed better in Category 1 than in Category 2 (all ps < .05). This implicated that our preservice teachers were more familiar with conventional computation skills than with informal operation skills in fractional procedural knowledge.

On the other hand, the preservice teachers also performed differently in the components of conceptual fraction knowledge ($F(7, 322) = 7.86$, $p < .001$), with a small contribution from the real differences of component means ($\eta^2 = .15$). Three categories of components were found in fraction conceptual knowledge, i.e., Category 1 (CON, TRN), Category 2 (EQU, ORD, ADD, SUB), and Category 3 (MUL, DIV). Category 1, the components of concept and transfer, referred to a general understanding of fraction. Category 2, the components of equivalence, order, addition, and subtraction, referred to a basic understanding of fractional computation. Category 3, the components of multiplication and division, referred to a complicated understanding of fraction. In our analysis, the preservice teachers performed significantly best in Category 1, then Category 2, and then Category 3 (all ps < .05). It was the same as Ball (1990) indicated that division of fractions were the most difficult understandings for preservice teachers.

Recommendations

In this study, preservice teachers' problem solving ability correlated with their fraction procedural knowledge. We also found the preservice teachers performed better fraction knowledge on procedure than on conception, and performed differentially in different fraction components, regarding to respectively procedural and conceptual knowledge. From the findings, it implicated a necessity for enriching preservice teachers' fraction knowledge, especially their fraction conceptual knowledge. Most preservice teachers in Taiwan received fraction knowledge through computational or procedural operations. The superiority of performances on fraction procedural knowledge might become their obstruction of understanding fraction. They need more stimuli to construct their conceptual knowledge, especially when they will be teaching mathematics in elementary schools in two or three years later.

In addition, there still are some limitations to this study. First is the small sample size. The authors only analyzed the participants enrolled in education program at one university. A repeated study with a larger sample size could enhance the validity of the results if the second study results were similar. Second, the authors instructed a mathematics course for these preservice teachers by using an open approach (Hashimoto & Becker, 1999). This might influence preservice teachers' performances on fractional understanding. But it is also a good source for investigating the effect of open-approach instruction in future.

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