Black swans and VaR

Michael Adams
Jacksonville University

Barry Thornton
Jacksonville University

ABSTRACT

There are two general classes of probability domains; each is very distinct, both qualitatively and quantitatively. The first distribution is referred to as a "Gaussian-Poisson distribution" – thin tail, the second, a “fractal” or Mandelbrotian distribution - fat tail. In thin-tail distributions, statistical exceptions occur but they don't carry unusually large consequences. In fat-tail distributions, when significant deviations (black swans) occur, the consequences are usually catastrophic in nature. Standard deviation is a useful statistical measurement of risk, if the underlying asset returns are distributed in a normal fashion about the mean. However, if the asset returns deviate significantly from what would be expected in a Gaussian distribution, standard deviation is often an inadequate and poor measurement of total risk and can often result in the serious underestimation of potential losses. Albeit, VaR is fairly accurate in predicting small daily losses with high probability, it breaks down completely in forecasting large catastrophic losses that exist in the tail of the distribution – black swan events. A glance at the Dow Jones Industrial Average's biggest one-day gains and losses confirms the existence of black swan events: On each of two different days in October 2008, the Dow surged more than 10%. On each of eight different days in late 2008, the Dow gave back more than 5%. 2008 was a particularly volatile year, but big moves can happen any time. In this article, the theoretical normality assumption is empirically tested (and clearly rejected) using time series of stock returns for the U.S. stock market for the last 30 years. The purpose of this study is to illustrate the existence of black swan events and their historical frequency, paying particular attention to the time period 1980-2010.

Keywords: VaR, Black Swans, Mandelbrotian distribution, kurtosis, leptokurtosis, fat tails

BACKGROUND

The term “Black Swan” comes from the common misconception that 'All swans are white' and has found general application in the financial markets, especially in light of the 1987 stock market crash , the 2008/09 credit market meltdown and the 2010 flash crash. The prevalent belief held among 17th century Northern Europeans that there was no such thing as a black swan, since no one had ever actually seen one. However, the discovery of the existence of black swans (*Cygnus atratus*) in Australia transformed the term to connote that a perceived impossibility had actually occurred. A Black Swan is a metaphor coined by Nasim Taleb to describe events that are apparently possible, but could *not* have been predicted based on past evidence. Taleb notes people’s heuristic to disbelieve that which one cannot predict. Taleb (2007), states “although these unpredictable deviations are fairly rare, they cannot be dismissed as outliers because cumulatively, their impact is so dramatic." (p. 27).

Before Taleb, David Hume (1748), John Mill (1843) and Karl Popper (1968) all described the problem of drawing general conclusions from limited observations. In general, financial decision models are generally based on a model predicting possible financial outcomes not only often ignores, but always limits the impact of events which are considered outliers to the model. The point is, a single such sighting of a black swan can invalidate a general statement (i.e., “all swans are white”). Taleb however was not the first to discover “fat tails” in stock returns. For example, Benoît Mandelbrot (1963) posited in a seminal article that changes in cotton prices could generally “be modeled by a stable Paretian distribution with a characteristic exponent less than 2.” (p. 25). Mandelbrot tested the infinite-variance hypothesis (fat tails) by calculating the standard deviation of changes in the prices on the cotton exchange and concluded that the return variances appeared to be much more erratic than what would be normally expected under an infinite-variance hypothesis.

INTRODUCTION

Modern financial theory has advanced our understanding of financial markets immensely; albeit, some of financial theory’s foundational assumptions do not appear to be borne out by market realities. Many financial models in neo-classical finance theory are predicated on the premise that changes in stock returns are normally distributed around the mean in the well-known bell curve. A normal distribution is a powerful analytical tool, because one can specify the distribution with only two variables, the mean and the square root of the variance. However, these models are remiss in capturing ‘fat tails’: infrequent but very large price changes. Fat tails are closely related to power laws, a mathematical link between two variables that are characterized by frequent small events and infrequent large events. (Mauboussin, 2002). This is the result of the combination of kurtosis risk and the risk associated with skewness. The overall returns can be dominated by extreme events (kurtosis), which are skewed to the downside. These fat-tail distributions pose several fundamental risk management problems. These problems include: (1) the presence of extreme adverse events, (2) some random unobserved events, and (3) hard-to-compute expectations.

Assumptions of normal distributions of stock returns are the pillars of finance models, including modern portfolio theory (mean-variance criterion), the capital asset pricing, Value-at-Risk (VaR), and the Black-Scholes models. The 1987 stock market crash was so improbable (black Monday was a black swan - 18 sigma event) given the standard statistical models used in finance, that it has called the entire basis of neo-classical finance models into question. This has led many to conclude that there are some recurring events, perhaps one or two per decade, that overwhelms the statistical assumptions embedded in the standard finance models employed for trading, investment management and derivative pricing (Estrada 2008)**.** These statistical anomalies appeared to affect many financial markets at once, including ones that were normally not thought to be correlated. Moreover, these severe market events seldom had a discernible economic cause or warning. These rare events were later named "Black Swans" by Taleb (2007) and the concept has extended far beyond finance. By definition, black swan events lack the historical perspective needed to perform *ex-ante* mathematical risk analysis.

REVIEW OF LITERATURE

Bachelier (1900) was the one of the first mathematicians to discover that most asset returns tend to follow a random walk or Brownian motion. Mandlebrot (1963) suggested that stock returns were in fact leptokurtic and thereby rejected the Gaussian distribution hypothesis. In 1964, Cootner also concluded that the random nature of stock returns makes the use of standard deviation a relatively poor proxy for risk assessment. Starting with Engle (1982) and later with Bollerslev in 1986, it has been shown that “Generalized Autoregressive Conditional Heteroskedasticity models” also referred to as GARCH models more accurately describe the random nature of actual stock returns than the Gaussian distribution assumption. Benoit Mandlebrot concluded that on the basis of his study of stock returns, that the over reliance on the assumption of normal distribution of return has resulted in serious flaws in most modern financial models including the Black Scholes option pricing model, and Benoît Mandelbrot, a French mathematician, extensively researched this issue. He concluded that the extensive reliance on the normal distribution for much of the body of modern finance and investment theory is a serious flaw of finance modeling (including the Black-Scholes option model, and the Capital Asset Pricing Model). He explained his views and alternative finance theory in a book: *The Misbehavior of Markets*. Mandelbrot (1997) proposed to replace the normality assumption with a “fractal view of risk, ruin, and reward.” (p. 67). According to the fractal view, large stock return swings are far more clustered than what would be predicted if the market followed a random walk normal distribution; the prevailing paradigm in investments. Later, Eugene Fama demonstrated that extreme returns occurred with much greater frequency than if returns were normally distributed by testing stock price changes. According to Fama (1965):

> “Mandelbrot is right. The distribution is fat-tailed relative to the normal distribution. In other words, extreme returns occur much more often than would be expected if returns were normal. As the result, the normality of stock returns has been replaced by the assumption of fat-tailed distributions in a wide variety of markets, assets, and time periods.” (p. 78).

Kon (1984), Berglund and Liljeblom (1990), Campbell and Hentschel (1992), Chan and Lakonishok (1992), and later, Frennberg (1993), after examining foreign markets covering different time periods, all concluded that stock returns tend to be leptokurtically distributed, not Gaussian in nature. Roll (1988) posits that the very nature of stock market returns to quickly reflect new information (EMH) tends to increase the degree of kurtosis of the returns and often results in increased outliers or fat tails when compared to the ubiquitous normal distribution. This does not address the question of why black swan events appear to cluster together.

METHODOLOGY

A standard normal distribution is a normal bell shaped distribution with a unit variance ($\sigma^2 = 1$) and a zero mean ($\mu = 0$). A Mandelbrot distribution is a probability distribution in which there is a *large* probability of experiencing a *small* gain, coupled with a *small* probability of experiencing a very *large* loss; which more than outweighs the gains. The mean of this theoretical distribution is less than 0 and the variance is infinite. The empirical distribution has fat tails (leptokurtic) and a high degree of kurtosis as compared to the normal distribution.
A return distribution that is very peaked or tightly distributed exhibits a high degree of kurtosis and usually will be characterized by fat tails in the distribution. This is especially true when compared with the ubiquitous thin tailed normal distribution. These fat tail mean that there exists a greater probability for extreme events occurring and are referred to in the literature as leptokurtosis. To determine the frequency of black swan events occurring, a thirty year time period was selected utilizing daily, weekly and monthly returns. The s&p500 serves as the proxy for the market. (Source: Yahoo).

To empirically identify the presence of extreme black swans in the U.S. stock markets, two different methodologies were used in this study in an attempt to empirically identify the presence of extreme black swan events on the NYSE. The first method was to calculate the statistical properties of stock returns on a monthly, weekly and daily basis. Secondly, the ranges of distribution frequencies were computed for daily, weekly and monthly returns over the time period of March 31, 1979 and March 31, 2009.

VALUE AT RISK

Value at Risk (VaR) has been widely employed to ascertain the potential risk for losses on a financial asset portfolio and is usually defined as the maximum value that the “mark-to-market” loss probability on the portfolio will exceed this threshold value This is often referred to as the potential loss to asset values measured in the firm’s daily accounts. VaR assumes a log-normal diffusion return process based upon an underlying normal distribution of returns. A loss which exceeds the VaR threshold is generally termed a “VaR break,” with a probability level of one minus the statistical probability of a VaR break occurring. VaR simply marks the statistical boundary between normal days and extreme event (black swan) days.

NORMAL VS. FAT-TAIL VAR

For a probability distribution to be considered stable, all the independent random variables must also have the same distribution as the constants alpha and beta. Moreover, the normal distribution is the only stable distribution whose standard deviation is defined; all other distributions have standard deviations that are either infinite or undefined. The general class of stable distributions was first identified by Levy in 1998

According to Wikipedia: "Given some confidence level $t\alpha \in (0,1)$ the VaR of the portfolio at the confidence level α is given by the smallest number l such that the probability that the loss L exceeds l is not larger than (1 – α)" (source: Wikipedia (2009). A normal distribution is defined as a distribution whose expected value = 0 and whose sigma = 1. Ergo, most theoretical research performed is usually presented in characteristic function instead of probability density functions as is usual with normal probability distributions. More precisely, the distribution of any random variable (X) is said to contain a fat tails if the following condition is met:

$$\Pr[X > x] \sim x^{-(1+\alpha)} \text{ as } x \to \infty, \qquad \alpha > 0.$$

By contrast to fat tail distributions, the normal distribution posits events that deviate from the mean by five or more standard deviations ("5-sigma event") are extremely rare, with 10- or more sigma being practically impossible. Fat tail distributions are examples of fat tail distributions that have "infinite sigma" (more technically: "the variance does not exist").
The rationale for adopting the normality distribution assumption was to not reduce the theoretical model to a mere empirical distribution fitting, Mandelbrot thought it was foolish to have to assume different distributions for daily, monthly and annual stock returns. Instead, he sought a model that could be applied to all classes of asset returns, regardless of the time interval being measured. He found that he could accomplish this by assuming a normal distribution of returns (Mandelbrot, 1999). Thus when return data naturally arise from a fat tail distribution, if the normal distribution model of risk is assumed, then an estimate of the corresponding sigma based necessarily on a finite sample size, severely understates the actual risk. Many academicians have noted this shortcoming of the normal distribution model and have proposed that fat tail distributions such as the stable distribution govern asset returns to replace the normality assumption in financial models (Mandelbrot 2008; Taleb 2009).

Most distributions that are considered "stable" usually contain "fat tails". The actual S&P500 return distribution demonstrated much greater volatility than what would be expected based upon a priori probability. Chart 1 shows what the expected normal frequency distribution of returns one would expect from 1979-2009 (dashed line) on the 'normal' distribution. The actual realized returns on the market index (SPX) is (1) highly right skewed, (2) much larger frequency of returns (leptokurtosis) occurring around the mean. The frequency of returns that occurred between 1 and 2 standard deviations was much smaller than expected and (3) finally very large negative returns occurred much more frequently than predicted. In this context, 'fat tails' simply refers to the larger -than-expected large positive or negative returns.

ANALYSIS

The basic research question is: Did the samples come from normally distributed populations? Null Hypothesis: There is no statistically significant difference between the expected and observed frequencies. If there are no differences then the null hypothesis is rejected and it can be concluded that the sample in question was drawn from a normally distributed population. Alternative Hypothesis: There is a difference between the observed and expected frequencies. The sample was not drawn from a normally distributed hypothesis. A normal distribution is expected to have a 0 kurtosis, with a stable distribution expected to have a 1 kurtosis. Negative skewness indicates that the curve has a noticeable longer left (or negative) tail. The distribution does not appear to log normal either, but appears to be more representative of a fat tail distribution. See table 1.

The SP500 fell over 23% in a single day on October 19, 1989. Then the U.S. stock market grew for an entire decade without experiencing any extreme fat-tail days (+5sigma). Even the day after September 11 terrorist attacks on NY, the stock market fell by only 4.9%, (a -4σ daily return). This "quiet" market environment appeared to change significantly starting in 2008. As shown in Table 2, there was a significant frequency increase in the observed number of actual black swan days. For example, On September 29th, 2008 the stock market suffered the first and only $1 trillion dollar one day loss of wealth. The frequency of actual black swan days occurred during the period under study is shown in Table 2. In the thirty year period under study, there were five times (5X) the 3.4% daily price change threshold and 97 times (97X) the 4.5% daily price change threshold predicated. A black swan day is defined as +/- 3 standard deviations away from the mean. Deviations should occur with certain frequencies; the larger the standard deviations results in greater variance and thus greater risk. There is a reverse correlation between the level of volatility and the expected frequency associated with the event.

OBSERVATIONS

Under the statistical normal distribution assumption of performance returns, deviations from the mean return should occur with a certain frequency; the greater the deviance, the lower the frequency associated with occurring Table 4 shows the daily performance of the S&P 500 relative to what would be predicted based upon the analysis. Clearly the probability for significant negative returns, which is commonly referred as three standard deviations from the mean (99.9%), exceeds the probability based on the model output.

Employing a Chi-Square goodness-of-fit test, the p-Value of 0.000 indicates that there is no chance that the observations come from a normal distribution. See table 3. In Table 3, the p-value corresponding to the computed value of the chi-square test statistic is 0.000. The null hypothesis is strongly rejected. The statistical evidence suggests that there is virtually no chance that these test samples were selected from a normally distributed population.

The distributions are "pinched" around the mean. In other words, the actual distributions are not mesokurtic / thin tailed like the normal, they are leptokurtic / fat tailed. The appropriate theoretical probability distribution to use in a model (like VaR) would have to be leptokurtic (very peaked in the center, pinched around the mean, and fat tails) and skewed left. Figure 4 shows the number of days in a particular year where the daily percent return was more than 3 standard deviations below the mean. In the time period under investigation, there were 53 such days. Note that more than a third of these "bad days" took place in 2008, and another 13 percent of them took place in 2009. In fact, 2008 and 2009 combined account for 26 of the 53 "bad days." This is almost half of them. See Figure 2

The expected values were determined by multiplying the number of observations in the sample by the following values which were derived from the normal distribution: It is interesting to note the volatility in returns for 2008 and 2009 with 26/53 3+/- standard deviation day movements occurred in this time period. The clustering effect is clearly evident. See Figures 4,5,6. Notes on Q-Q Plots (Figures 5, 6 and 7). A Q-Q plot can be employed to check the assertion that a variable is normally distributed. If the variable is normal, the plot of points will lie along the line. Notice that the horizontal axis measures the percent return and the vertical axis represents the percentile. Note how the vertical axis is stretched at the ends and then packed around the center – this is because you're making a nonlinear relationship linear. The most striking thing about the Q-Q plots is that you can see that the variables (daily, weekly, and monthly S&P 500 percent returns) are not normal, and in particular there is a strong deviation from normal in the tails – and in particular, the left side tail.

CONCLUSIONS

A VaR model typically assumes a log-normal price diffusion process, and that the log-return process follows a normal distribution. However, real financial markets exhibit several deviations from this ideal, albeit, useful model. The market distribution for stocks has several realistic properties not found in the prevalent log-normal models. Today, most financial models measure VaR based on the thin-tailed and symmetric normal, "bell-shaped" distribution curve. As demonstrated by the stock market meltdown of 2008, these normal distribution assumptions resulted in overly optimistic VaR estimates; inadequately accounting for extreme events.

The purpose of this paper was to empirically examine the underlying stock return distribution for the past thirty year period. In 2008, most financial strategies/models were predicated on the flawed assumption of normally distributed returns demonstrated how fat-tail risk can wipe out an investor. For example, Bear Sterns, Fanny, Freddy, Merrill Lynch and AIG all succumbed to the black swan market credit market event of 2008/09 and subsequently failed because of their exposure to certain fat tail risks. It appears that professional investors have become much better at understanding and managing predictable portfolio risks based on assumed probabilities of recent financial hedging tool that have been recently developed. Unfortunately, the recent financial history has also witnessed financial events that could not be predicted based on prior events and have often been quite severe and have resulted in large losses being posted.

Unfortunately, the recent financial history has witnessed a number of extreme, and often severe, events that could not be predicted based on prior events. It would appear that modern finance techniques and tools may in fact, be making the tails of the distribution much fatter. By hedging idiosyncratic risk, such as foreign exchange rates, interest rates, commodity prices and so forth, one can make their portfolio appear to be relatively safe. However, swapping everyday risk for the exceptional risk results in these 'black swan' events occurring much more frequently than conventional theory would suggest. As a result, financial markets today appear to be plagued not only by 'black swans' but also by pernicious "gray" swans that occur a lot more frequently than expected.

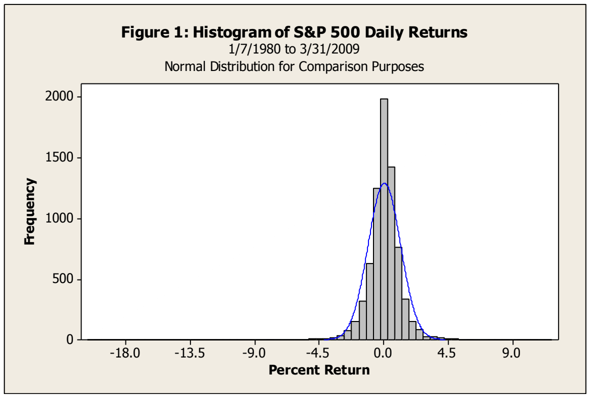
SUMMARY

If the observations do not appear to be symmetrically spread around the expected value, the distribution is said to be skewed. If the distribution is skewed, the average and the median return will be quite different. If one assumes that the variables are symmetrical about the mean, thereby ignoring skewness risk, the results of the financial model will substantially understate the risk/return characteristics. Kurtosis risk will cause any model to understate the risk of variables with high kurtosis. For instance, Long-Term Capital Management (LTCM) ignored kurtosis risk to its detriment. LTCM had to be bailed out by major investment banks in the late 90s because it understated the kurtosis of many financial securities underlying the fund's own trading positions. If skewness risk and kurtosis risk is ignored, the resultant conclusions based on the VaR model will be subjected to serious estimation error.

The VaR measure should be considered as an insufficient risk performance measure, since it ignores *both* the skewness and kurtosis of the return. It also ignores all the fractional moments resulting from the long-term interdependencies of stock market price changes/returns. As a result, the VaR methodology gives far too little importance to extreme outliers that when they occur are not only catastrophic in nature but also appear to cluster. This is not limited to VaR however; the Black-Scholes model of option pricing is also predicated on a normal distribution. If the stock return distribution is actually a fat-tailed one, then the model will under-price options that are far out of the money, since a 5 or 7 sigma event is more likely than what could be expected under a Gaussian distribution or bell shaped curve.

In summary, small price changes appear more frequently and large negative changes appear far greater than what a normal distribution would predict. Greater than-expected large price changes occurred with greater frequency of these events increased significantly in 2008 and 2009. As a result, there exists much fatter tails with higher probabilities and far less medium sized changes than the normal distribution model would suggest under the standard risk models being employed thus. Extremely large stock price changes appear to have now become the new norm post 2008. As a result, VaR fails to adequately measure the tail beyond the 99% confidence interval leaving out some reasonably uncommon, but extremely large potential for financial losses due to the existence of black swan events; exactly the kind of losses that portfolio risk managers should be most concerned about. Black swans appear to be alive and well, both in nature and in the capital markets.

Chart 1: Histogram



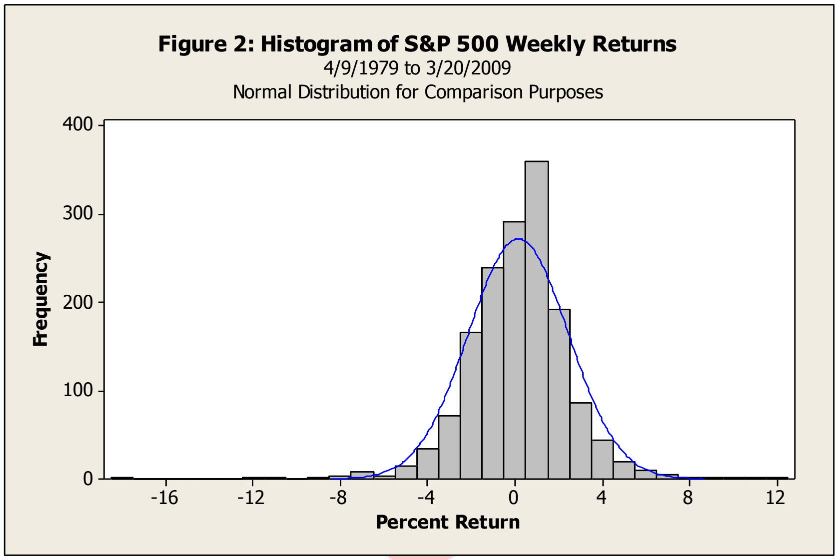


Figure 3: Histogram of S&P 500 Monthly Returns
5/1/1979 to 3/2/2009
Normal Distribution for Comparison Purposes

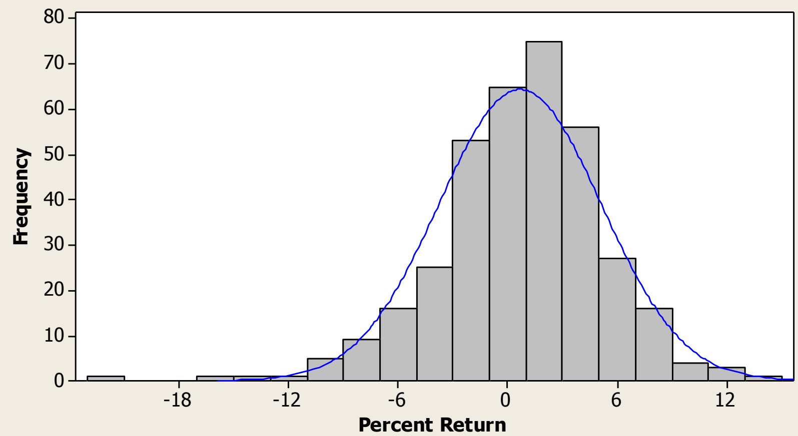


Table 1: Descriptive Statistics for S&P 500 Returns

	Daily Returns	**Weekly Returns**	**Monthly Returns**
Dates	4/9/79 – 3/31/2009	4/9/1979 – 3/20/2009	4/1/1979 – 3/2/2009
Number of Observations	7377	1564	359
Mean	0.0338%	0.1574%	0.676%
Median	0.0483%	0.3049%	1.015%
Standard Deviation	1.1376%	2.2935%	4.447%
Skewness	**-0.82**	**-0.50**	**-0.70**
Kurtosis	**22.79**	**5.31**	**2.36**
Maximum	**11.58%**	**12.0258%**	**13.177%**
Minimum	**-20.4669%**	**-18.1955%**	**-21.763%**

Table 2: Number of Daily S&P 500 Percent Returns More Than 3 Standard Deviations Below the Mean

Year	Number of Days	Percent
1982	1	1.89
1983	0	0.00
1984	0	0.00
1985	0	0.00
1986	1	1.89
1987	6	11.32
1988	2	3.77
1989	1	1.89
1990	0	0.00
1991	1	1.89
1992	0	0.00
1993	0	0.00
1994	0	0.00
1995	0	0.00
1996	0	0.00
1997	1	1.89
1998	3	5.66
1999	0	0.00
2000	2	3.77
2001	3	5.66
2002	4	7.55
2003	1	1.89
2004	0	0.00
2005	0	0.00
2006	0	0.00
2007	1	1.89
2008	**19****	**35.85**
2009	**7***	**13.21**
Total	53	100.00

Black Swan events 1916-2010:

Daily change + or –	Expected	Actual	Actual vs. Expected
>3.4%	81	425	5x
>4.5%	2	194	97x
>7.0%	1 in 75,600,000	44	

Table 3: Results of Chi-Square Analysis of S&P Returns

Range of Z Scores	Observed Frequencies Daily	Expected Frequencies Daily	Observed Frequencies Weekly	Expected Frequencies Weekly	Observed Frequencies Monthly	Expected Frequencies Monthly
$Z \leq -3$	53	9.96	15	2.11	3	0.48
$-3 < Z \leq -2$	118	157.87	23	33.47	10	7.68
$-2 < Z \leq -1$	596	1002.61	153	212.56	32	48.79
$-1 < Z \leq 0$	2856	2518.07	543	533.86	114	122.54
$0 \leq Z < 1$	3001	2518.07	651	533.86	153	122.54
$1 \leq Z < 2$	595	1002.61	143	212.56	40	48.79
$2 \leq Z < 3$	101	157.87	26	33.47	7	7.68
$Z \geq 3$	57	9.96	10	2.11	0	0.48
Total	7377	7377	1564	1564	359	359
χ^2 = 907.35		**P-Value = 0.000**	**χ^2 = 178.41**	**PValue = 0.000**	**χ^2 = 29.83**	**P-Value = 0.000**

Figure 4: Q-Q Plot for Daily S&P 500 Returns
Normal

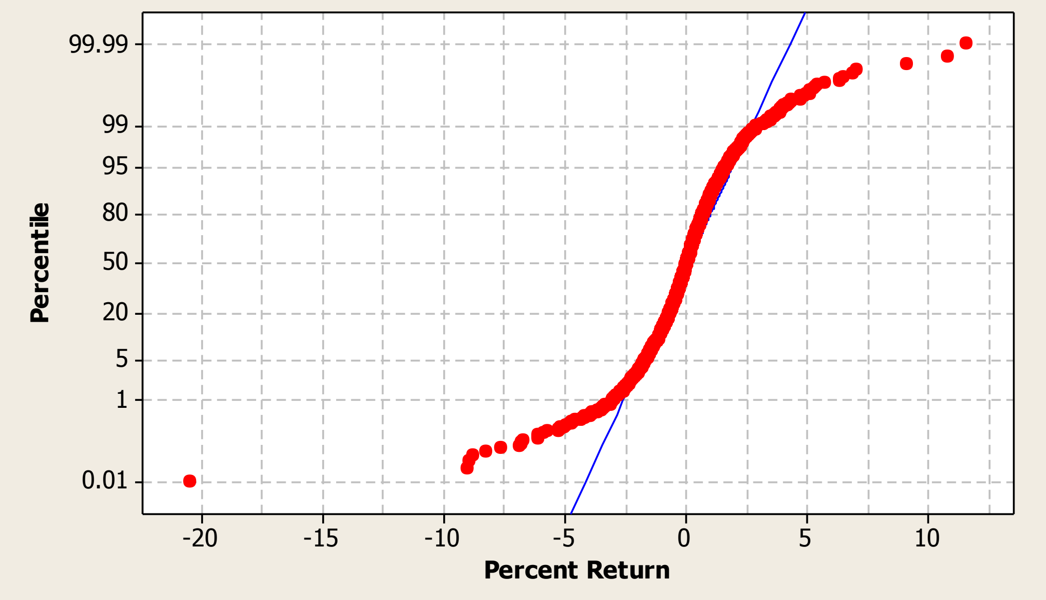


Figure 5: Q-Q Plot for Weekly S&P 500 Returns
Normal

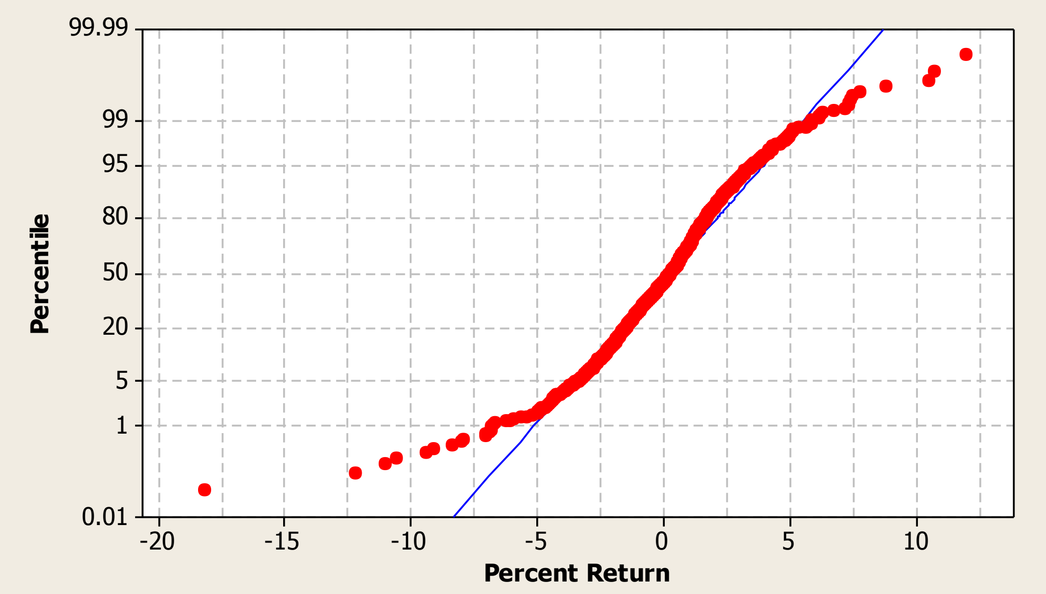


Figure 6: Q-Q Plot for Monthly S&P 500 Returns
Normal

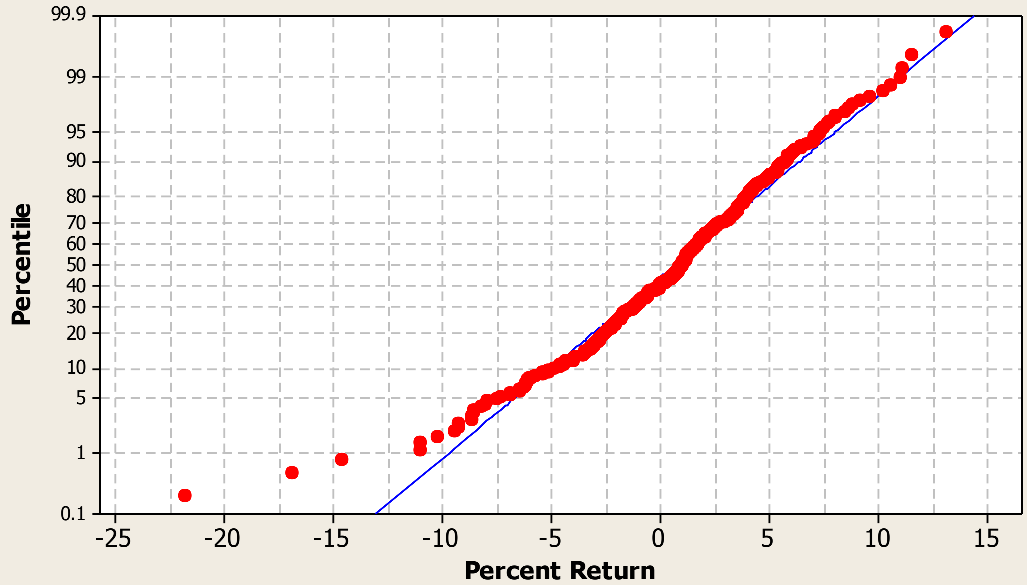
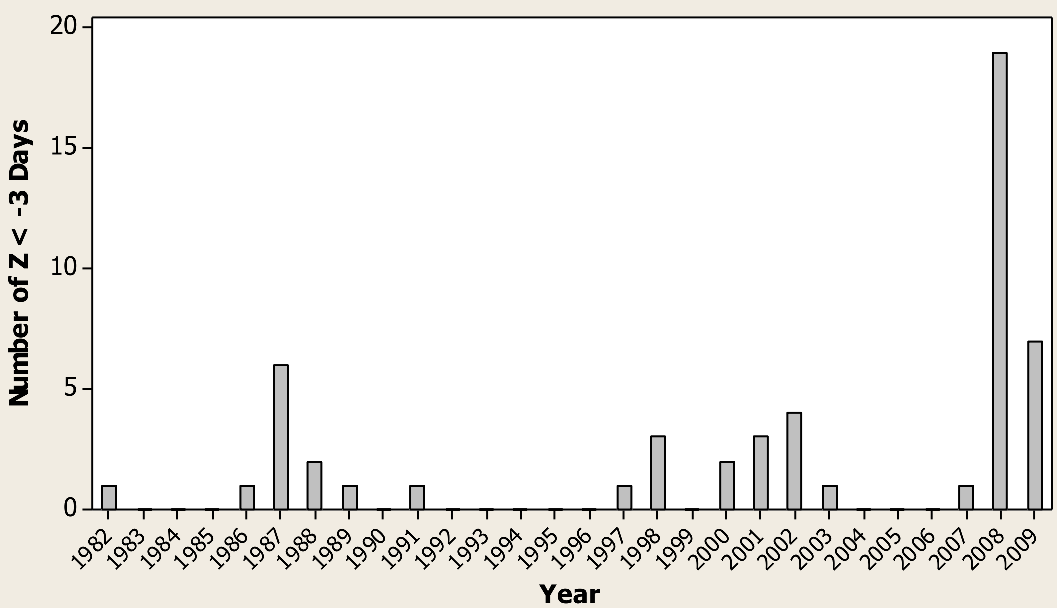


Figure 7: Number of Daily S&P 500 Percent Returns More Than 3 Standard Deviations Below the Mean



REFERENCES

Alexander, Gordon J. (2009). From Markowitz to modern risk management, *The European Journal of Finance*, Retrieved June 30, 2009.

Ang, Andrew (2006). "The cross-section of volatility and expected returns," *Journal of Finance*, Vol. 61(1), pp. 259–299.

Aparicio, L and P. Estrada (2001). Empirical distributions of stock returns: European securities markets, 1990-95. RePEc:taf:eurjfi:v:7:y:2001:i:1:pp:1-21.

Bachelier, Louise (1900). "*Théorie de la Spéculation, Annales de l'Ecole normale superior,"* (trans. Random Character of Stock Market Prices).

Berglund, T., and E. Liljeblom (1990), **"**The covariance-factor structure of daily returns in a thinly traded stock market," *Journal of Multinational Financial Management*, Vol. 7(2), June, pp. 113-125.

Berkowitz, J (2009). "Evaluating value-at-risk models with desk-level data," *Management Science*, January, 2009.

Bikhchandani, Sushi and Sunil Sharma, (2001). "Herd behavior in financial markets," I *Journal of Business Finance & Accounting*, 11/2004. Vol. 31(9-10), pp. 1389–1417.

Brown, Aaron, (1997). "The next ten VaR disasters," *Derivatives Strategy*, March, 1997. Vol. 13(1), February, pp. 39-55.

Chan, L.K.C., and J. Lakonishok (1992). "Robust measurement of beta risk," *Journal of Financial and Quantitative Analysis* 27, pp. 265-282.

Chen, Ren-Raw and Palmon, Oded. (2005), "A non-parametric option pricing model: theory and empirical evidence,*" Review of Quantitative Finance and Accounting*, March Vol. 24(2), pp. 115-134.

Claudio, Gallo (2010). "Mathematical Models of Financial Markets; Application of Math in Models." Information System and Neural Networks." Information System Survey, May 2010.

Collins, Daniel P. (2009). "Watch out for those fat tails: how could so many so-called diverse trading strategies do so poorly?" *Futures* (Cedar Falls, Iowa), Vol. 38 April, pp. 18-37.

Cootner, Paul H. (1964). *The Random Character of Stock Market Prices*, Boston, Mass. MIT.

Crouhy, Michel, Dan Galai and Robert Mark, (2001) *The Essentials of Risk Management*. McGraw-Hill.

Degiannakis, Stavros (2008). "Rolling-sampled parameters of ARCH and Levy-stable models," *Applied Economics*, Vol. 40(23), December, pp. 3051–3067.

Dowd, Kevin (2005). *Measuring Market Risk*. John Wiley & Sons.

Eling. Martin (2009). "Risk and return of reinsurance contracts under copula models," *The European Journal of Finance*, - 1351-847X PY – 2009 UR -

Embrechts. P. (2008). "Linear correlation and EVT; Properties and caveats," *Journal of Financial Econometrics*, Vol. 7(1), September, pp. 30-39.

Engle, Robert (1982). "Dynamic Conditional Correlation - A Simple Class of Multivariate GARCH Models," *Journal of Business and Economic Statistics* (July 2002), Vol. 20(3) pp. 27-42.

Estrada, Javier. (2008). "Black swans and market timing: how not to generate alpha," *Journal of Investing*, Fall, Vol. 17, pp. 117-165.

_____________. *(2002). "The temporal dimension of risk". Quarterly Review of Economics and Finance.*

Fama, Eugene (1965). “The Behavior of Stock Market Prices," *Journal of Business*, Vol. 38(1), pp. 34-105.

Fang, J.S. (2004). "Making the business case for process safety using value-at-risk concepts," *Journal of Hazardous Materials*, Vol. 115(1-3), November, pp. 17-26.

_______ (2007). "Value at Risk Perspective on Layers of Protection Analysis," *Process Safety and Environmental Protection, 7* Vol. 85(1), pp. 81-87.

Fong, Chan K. (2006). "Using extreme value theory to measure value-at-risk for daily electricity spot prices," *International Journal of Forecasting*, 200604/06. Vol. 22(2), April-June 2006, pp. 283-300.

Friedman. Jeffrey (2007). "Ignorance as a starting point: From modes epistemology to realistic political theory,” *Critical Review*, 01/2007. SN - 0891-3811 Vol. 19(1), pp 1-22.

Gencay, R. (2003). "High volatility, thick tails and extreme value theory in value-at-risk estimation," *Insurance Mathematics and Economics*, Vol. 33(2), October, pp. 337-356.

Gribbin, Donald W. Randy W. Harris and Hon - Shiang Lau (1992). "Futures prices are not stable-Paretian distributed," *Journal of Futures Markets*, Vol. 12(4) October, pp: 475-487.

Harmantzis, Fotios and Linyan Chien Miao, (2006). "Empirical study of value-at-risk and expected shortfall models with heavy tails," *Journal of Risk Finance*, Vol. 7, Spring pp.141-174.

Haas, M (2004). "Mixed normal conditional heteroskedasticity," *Journal of Financial Econometrics*, March, Vol. 2(2), pp. 211-250

Hasan, Zobaer, Kamil, Anton, Abdul Mustafa, and Baten, Azizul (2013). “An Empirical Analysis of Higher Moment Capital Asset Pricing Model for Bangladesh Stock Market”, *Modern Applied Science*

Haug, Espen (2007). *Derivative Models on Models*. John Wiley & Sons, 2007.

Hilliard, J.E. (1999). “Testing a jump-diffusion stochastic interest rates model in currency options markets”, Proceedings of the IEEE.IAFE Conference on computational intelligence for financial engineering.

Holton, Glyn (2003). *Value-at-Risk: Theory and Practice*, Academic Press, 2003.

Hume, David (1748). *A treatise of human nature*. McMillan and Co., New York, NY.

Jefferis, Keith (2005). “Capitalization and weak-form efficiency in the JSE securities exchange, *South African Journal of Economics.*

Jinlin, Ma (2010). “The Risk Measures based on GARCH model in the Tanker Shipping market,” *International Conference on Automation and Logistics*, 2010.

Jorion, Philippe (2006). *Value at Risk: The New Benchmark for Managing Financial Risk*, 3rd ed. McGraw-Hill. Kenney, J. F. and E. S. Keeping, (2001) *Mathematics of Statistics, Pt. 1, 3rd ed.* Princeton, NJ: Van Nostrand, pp. 129,134.

Kanellopoulou, Stella (2008). “Empirical distributions of stock returns: Paris stock market:. 1980-2003. *Applied Financial Economics*.

Kolman, Joe, Michael Onak, Philippe Jorion, Nassim Taleb, Emanuel Derman, Blu Putnam, Richard Sandor, Stan Jonas, Ron Dembo, George Holt, Richard Tanenbaum, William Margrabe, Dan Mudge, James Lam and Jim Rozsypal, Roundtable: The Limits of VaR. *Derivatives Strategy,* April 1998.

Kon, S.J., (1984). “Models of stock returns--a comparison,” *Journal of Finance* , Vol. 39(3), pp. 147-165.

Levy, Shiki (1998). "Wealthy people and fat tails: An explanation for the Lévy distribution of stock returns," November, *Journal of Finance.* Vol. 64(3), pp. 30-98
Liu, Mei-Ying Wu, Chi-Yeh Lee, and Hsien-Fen. (2004). "Fat--tails and VaR estimation using power EWMA models," *Journal of Academy of Business and Economics*, Vol. 3 January, pp. 114-167.
Longin, F.M. (2007). "From value at risk to stress testing: The extreme value approach," *Journal of Banking and Finance*, Vol. 24(7), July, pp. 1097-1130.
Los, Cornelis A., (2005) "Why VAR fails: Long memory and extreme events in financial markets". Vol. III (3), pp. 19-36.
_____________, (2003) "Managing VaR and extreme values", Routledge International Studies in Money and Banking.
Lowenstein, Roger (2000). *When Genius Failed: The Rise and Fall of Long-Term Capital Management* New York, Random House.
MacKenzie, Donald (2004). "The big, bad wolf and the rational market: portfolio insurance, the 1987 crash and the performativity of economics," *Economy and Society*, 1/1/2004. Vol. 33(3), August, pp. 303–334.
Mandelbrot, Benoit (1963). "The stable Paretian income distribution, when the apparent exponent is near two." *The Journal of Business*, Vol. 36(4), October, pp. 420-429.
_______________ (1997*). Fractals and Scaling in Finance: Discontinuity, Concentration, Risk.* New York, N.Y.Springer Verlag.
________________(2004). *The (Mis)behavior of Markets.* Basic Books, New York, NY
________________(1999). "A Multifractal Walk down Wall Street," *Scientific American*, February, pp.70-73.
Mauboussin. Michael J. (2002). "Revisiting market efficiency: The stock market as a complex adaptive system," *Journal of Applied Corporate Finance*, Vol. 14(4) pp: 47-55 2002 ON: 1745-6622 pp. 1078-1196.
_________________ (2008). *More than you know – Finding financial wisdom in unconventional places*. Columbia Business School Publishing, New York, N.Y.
McKelvey, Bill (2007). "Beyond Gaussian averages: redirecting international business and management research toward extreme events and power laws," *Journal of International* Vol. 38**,** December, pp. 1212–1230.
McNeil, Alexander, Rüdiger Frey and Paul Embrechts, (2005). *Quantitative Risk Management: Concepts Techniques and Tools*, Princeton University Press.
Mehla, S., and S.K. Goyal. (2012) "Empirical Evidence on Weak Form of Efficiency in the Indian Stock Market, *Asia-Pacific Journal of Management Research and Innovation*.
Milanovic, Vlade (2013) "Financial predictions using intelligent systems", Brunel University School of Engineering and Design PhD Thesis.
Marco, Corazza (1997). "Searching for fractal structure in agricultural futures markets*", Journal of Futures Markets.*
Mill, John Stuart (1874). *A System of Logic*, New York: Harper & Brothers, 1874.
Otranto, E. (2008). "Clustering heteroskedastic time series by model-based procedures," Computational statistics and data analysis," Vol. 52(10), 2008, pp. 4685-4698.
Pareto, Vilfredo (1999*). Critical Assessments*, London: Routledge, IV, pp. 236-240.
Pearson, Neil, (2002). *Risk Budgeting: Portfolio Problem Solving with Value-at-Risk*. John Wiley & Sons.

Peters, Edgar E. (1994). *Fractal Market Analysis* (New York: John Wiley & Sons, 1994), pp. 21-27.

Popper, K.R. (1968). "Epistemology without knowing subject," In Van Rootselaar/Staal, *Logic, Methodology, and Philosophy of Sciences*, pp. 333-373.

Rotfeld, Herbert J. (2007). "Theory, data, interpretations, and more theory (Editorial)," *Journal of Consumer Affairs*, Vol. 41(3), Winter pp. 131-136.

Scherer, Matthias (2011). "Minimally cross-entropic conditional density: a generalization of the GARCH model, Universitat Karlsruhe.

Soros, George (1995). *Soros on Soros* (New York: John Wiley & Sons), p. 17.

___________ (2008). *The New Paradigm for Financial Markets - The credit crisis of 2008 and what it means*, New York: Public Affairs.

Stavros, Degiannakis (2007). "Rolling-sampled parameters of ARCH and Levy-stable models," *Applied Economics*, Vol. 40(23), December 2008, pp. 3051–3067.

Sui, Wei (2007). "Quantitative methods in high frequency financial econometrics: modeling univariate and multivariate times series", Universitat Karlsruhe.

Taleb, Nassim (2007). *Fooled by Randomness: The hidden role of chance in the markets and life*. New York, N.Y.Random House.

_________ (1997). "The world according to Nassim Taleb," *Derivatives Strategy*, December 96/January 97.

_________ (2007). *The Black Swan: The Impact of the Highly Improbable* 2007. New

Wei, Yu, Wang Chen and Yu Lin (2013). "Measuring daily Value –at-Risk of SSEC index: A new approach based on multifractal analysis and extreme value theory", *Physica A Statistical Mechanics and its Applications.*

Wenjing, Qi (2010) "Dynamic Assessment and VaR based Information" *2nd International Conference of e business and Information System Survey*, May 2010.
York, NY. Random House. PROLOGUE pp. 8, 23-25.

Yu, Jing, Bin Xu, "The Strategic Asset Allocation Optimizer Model of Sovereign Wealth Funds" *Procedures of Computer Science,* 2010.

Zapedeau, Daniela (2012). "The Role of Value at Risk in the Management of Assets and Liabilities," Annals of University Orden, *Economic Science series*, 2012.